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2006 J. Phys. A: Math. Gen. 39 941

(http://iopscience.iop.org/0305-4470/39/4/014)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 941–949 doi:10.1088/0305-4470/39/4/014

Topological mass mechanism and exact fields mapping

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Received 2 September 2005
Published 11 January 2006
Online at stacks.iop.org/JPhysA/39/941

Abstract

We present a class of mappings between models with topological mass mechanism and purely topological models in arbitrary dimensions. These mappings are established by directly mapping the fields of one model in terms of the fields of the other model in closed expressions. These expressions provide the mappings of their actions as well as the mappings of their propagators. For a general class of models in which the topological model becomes the BF model the mappings present arbitrary functions which otherwise are absent for Chern–Simons like actions. This work generalizes the results of (Ventura O S, Amaral R L P G, Costa J V, Buffon L O and Lemes V E R 2004 *J. Phys. A: Math. Gen.* **37** 11711–23) for arbitrary dimensions.

PACS numbers: 11.10.Kk, 11.15.Tk

1. Introduction

The search for ultraviolet renormalizable models has always been one of the most attractive and relevant aspects of quantum field theory. As is well known, the program of describing the electro-weak interactions in the language of QFT is based on the construction of the Higgs mechanism for mass generation of the vector bosons. However, this mechanism relies on the existence of a scalar particle, the Higgs boson, whose experimental evidence is still lacking.

In this context, the topological mechanism for mass generation is attractive, since it provides masses for the gauge vector bosons without the explicit introduction of new scalar fields. For example, in three-dimensional spacetime, the topological non-Abelian Chern–Simons term generates mass for the Yang–Mills fields while preserving the exact gauge invariance [2]. Indeed an important property of the three-dimensional Yang–Mills type actions, in the presence of the Chern–Simons term, was pointed out in [3], i.e., it can be cast in the form of a purely Chern–Simons action through a nonlinear covariant redefinition of the gauge connection [4]. The quantum consequences of this fact were investigated in the BRST framework yielding an algebraic proof of the finiteness of the Yang–Mills action with topological mass [5].

In four dimensions the topological mass generation mechanism occurs in the case of an anti-symmetric tensorial field $B_{\mu\nu}$. It has been shown that the Cremmer–Sherk action gives a massive pole to the vector gauge field in the Abelian context [6]. Indeed, as was shown [7] this model exists only in the Abelian version. In fact, possible non-Abelian generalizations of the Cremmer–Sherk action will necessarily require non-renormalizable couplings, as in [8], or the introduction of extra fields [9]. Anti-symmetric rank-two fields in four dimensions also deserve attention since they appear naturally by integrating out the fermionic degrees of freedom in favour of bosonic fields in bosonization approaches. The fermionic current turns out to be expressed in terms of derivatives of a tensorial field as a topologically conserved current quite similar to the expression of the current in terms of vectorial fields in three dimensions [10]. The coupling of this current to the gauge field leads to terms in the effective action similar to the one of the Cremmer–Sherk model [11]. The mapping of Cremmer–Sherk's action to the pure BF action, in an iterative way, was presented in [12] both in the Abelian as well as in the non-Abelian cases. The exact forms of these mappings in the four as well as in the three-dimensional cases were presented in [1].

In D dimensions the topological mass generation mechanism occurs in a class of models with an anti-symmetric p-form field B coupled to a q-form field A, with $q + p = D - 1$. The interest for these models also appears from the bosonization procedure in arbitrary dimensions which is known to lead to antisymmetric tensorial fields with higher ranks [11].

In this work we generalize the results shown in [1] by presenting the exact mapping between generalized Cremmer–Sherk's actions in arbitrary dimensions and purely topological BF models. The closed expression we obtain depends on arbitrary functions and has a non-local nature. We also obtain the mapping from Maxwell–Chern–Simons like models and pure Chern–Simons ones in higher dimensions.

2. Generalized Cremmer–Cherk actions

The aim of this section is to establish the classical equivalence between the generalized Cremmer–Sherk's action and the pure BF theory, i.e., that the first action can be mapped to the second one through a redefinition of the antisymmetric fields. Following the same steps of the four-dimensional case [1], we search for a redefinition of the q-form gauge field A and the p-form gauge field B, subject to $q+p = D-1$, as closed expressions in terms of corresponding fields $\hat{A}$ and $\hat{B}$ in such a way that the equivalence of their actions can be asserted[6]:

$$\mathcal{S}_M(A; N_A) + \mathcal{S}_H(B; N_B) + \mathcal{S}_{BF}(A, B; N_{AB}) = \mathcal{S}_{BF}(\widehat{A}, \widehat{B}; \widehat{N}_{AB}). \tag{2.1}$$

Here

$$\mathcal{S}_M(A; N_A) = \int A \wedge d * dN_A A, \tag{2.2}$$

$$\mathcal{S}_H(B; N_B) = \int B \wedge d * dN_B B, \tag{2.3}$$

[6] We work in the Minkovski spacetime with $\eta_{\mu\nu} = \mathrm{Diag}(+, -, \ldots, -)$, $\varepsilon^{123\cdots D} = 1$ and $\varepsilon^{\mu_1\mu_2\cdots\mu_D} = \eta^{\mu_1\nu_1}\ldots\eta^{\mu_D\nu_D}\varepsilon_{\nu_1\nu_2\cdots\nu_D}$.

$$\mathcal{S}_{BF}(A,B;N_{AB}) = \int B\wedge dN_{AB}A, \tag{2.4}$$

where N_A, N_B, N_{AB} and $\widehat{N}_{AB}$ are generic scalar operators which commute with the operations $*$ and d. In the simplest case they will be just normalization factors. Indeed taking the field redefinitions in the form

$$A = \widehat{A} + C_{AA} * d * d\widehat{A} + C_{AB} * d\widehat{B}, \qquad B = \widehat{B} + C_{BB} * d * d\widehat{B} + C_{BA} * d\widehat{A} \tag{2.5}$$

and using this mapping in equation (2.1) the following set of equations are obtained:

$$\begin{aligned}
&N_B C_B^2 + C_{AB}^2 N_A\Delta - N_{AB}C_{AB}C_B = 0,\\
&N_B C_{BA}^2\Delta + N_A C_A^2 - N_A C_A C_{BA} \quad = 0,\\
&2N_B C_{BA}C_B\Delta + 2N_A C_A C_{AB}\Delta - N_{AB}C_{AB}C_{BA}\Delta - N_{AB}C_A C_B - \widehat{N}_{AB} = 0.
\end{aligned} \tag{2.6}$$

We have defined $C_A = 1 + C_{AA} * d * d$ and $C_B = 1 + C_{BB} * d * d$. Let us mention that the action of the delta operator on a p-form w is given by

$$\Delta w = (-1)^{Dp}(d * d * + (-1)^D * d * d)w = -g^{\mu\nu}\partial_\mu\partial_\nu w.$$

Note that Δ commutes with the operations $*$ and d. Also for forms w and v with compact support it follows that

$$\int w\wedge\Delta v = \int \Delta w\wedge v. \tag{2.7}$$

This means that the operator Δ can be dealt with as if it were a scalar. The same properties will be warranted to the coefficients C_{XY} (with X and Y meaning A and/or B).

The coefficients that solve equations (2.6) turn out to be

$$\begin{aligned}
C_{AA} &= \frac{(-1)^{D(D-p)}\widehat{N}_{AB}^{1/2}}{\Delta}\left[\frac{2^{1/2}}{\sigma}\left\{\frac{N_{AB} + \varepsilon\left[N_{AB}^2 - 4N_AN_B\Delta\right]^{1/2}}{\left(N_{AB}^2 - 4N_AN_B\Delta\right)}\right\}^{1/2} - \frac{1}{N_{AB}^{1/2}}\right],\\
C_{BB} &= \frac{(-1)^{D(p+1)}\widehat{N}_{AB}^{1/2}}{\Delta}\left[\frac{\sigma}{2^{3/2}}\left\{\frac{N_{AB} + \varepsilon\left[N_{AB}^2 - 4N_AN_B\Delta\right]^{1/2}}{\left(N_{AB}^2 - 4N_AN_B\Delta\right)}\right\}^{1/2} - \frac{1}{N_{AB}^{1/2}}\right],\\
C_{AB} &= \frac{\sigma\widehat{N}_{AB}^{1/2}}{2^{3/2}N_A\Delta}\left\{\frac{N_AN_B\Delta\left(N_{AB} - \varepsilon\left[N_{AB}^2 - 4N_AN_B\Delta\right]^{1/2}\right)}{\left(N_{AB}^2 - 4N_AN_B\Delta\right)}\right\}^{1/2},\\
C_{BA} &= \frac{2^{1/2}\widehat{N}_{AB}^{1/2}}{\Delta\sigma N_B}\left\{\frac{N_AN_B\Delta\left(N_{AB} - \varepsilon\left[N_{AB}^2 - 4N_AN_B\Delta\right]^{1/2}\right)}{\left(N_{AB}^2 - 4N_AN_B\Delta\right)}\right\}^{1/2}.
\end{aligned} \tag{2.8}$$

Here σ are operators we discuss in the following while ϵ is inserted just to keep control of the branches of the square root.

Looking for the inverse mapping we search for $\widehat{C}_{XY}$ such that

$$\widehat{A} = A + \widehat{C}_{AA} * d * dA + \widehat{C}_{AB} * dB, \qquad \widehat{B} = B + \widehat{C}_{BB} * d * dB + \widehat{C}_{BA} * dA. \tag{2.9}$$

Using (2.5) in (2.9), we find after some manipulations

$$\begin{aligned}
\widehat{C}_{AA} &= \frac{(-1)^{D(D-p)}}{\widehat{N}_{AB}^{1/2}\Delta}\left[\frac{2^{1/2}}{\widehat{\sigma}}\left\{N_{AB}+\varepsilon\left[N_{AB}^2-4N_AN_B\Delta\right]^{1/2}\right\}^{1/2}-N_{AB}^{1/2}\right],\\
\widehat{C}_{BB} &= \frac{(-1)^{D(p+1)}}{\widehat{N}_{AB}^{1/2}\Delta}\left[\frac{N_A\widehat{\sigma}}{2^{1/2}}\left\{N_{AB}+\varepsilon\left[N_{AB}^2-4N_AN_B\Delta\right]^{1/2}\right\}^{-1/2}-N_{AB}^{1/2}\right],\\
\widehat{C}_{AB} &= \frac{N_B2^{3/2}}{\widehat{\sigma}\widehat{N}_{AB}^{1/2}}\left\{N_{AB}+\varepsilon\left[N_{AB}^2-4N_AN_B\Delta\right]^{1/2}\right\}^{-1/2},\\
\widehat{C}_{BA} &= \frac{\widehat{\sigma}}{2^{3/2}\widehat{N}_{AB}^{1/2}}\left\{N_{AB}+\varepsilon\left[N_{AB}^2-4N_AN_B\Delta\right]^{1/2}\right\}^{1/2}.
\end{aligned} \tag{2.10}$$

Observe that the presence of the arbitrary operator σ or $\widehat{\sigma}$ in equations (2.8) and (2.10) has been noted in the four-dimensional case [1]. It is expected since the set of transformations

$$\hat{A}\longrightarrow\sigma\hat{A},\qquad \hat{B}\longrightarrow\frac{1}{\sigma}\hat{B}, \tag{2.11}$$

does not affect the generalized BF model action.

Let us now investigate the usual Cremmer–Sherk model, that is, the case in which the arbitrary operators present in the actions are reduced to normalization factors that can be fixed as

$$N_{AB}=(-1)^pm, \tag{2.12}$$

$$N_A=N_B=\frac{(-1)^{D(p+1)+1}}{2}. \tag{2.13}$$

In this case the mapping is defined by

$$\begin{aligned}
C_{AA} &= \frac{4(-1)^{D^2-1}}{\sigma^2}C_{BB}=\frac{(-1)^{D(D-p)}}{\Delta}\left[\frac{(2m)^{1/2}}{\sigma}\left[\frac{m\pm\sqrt{m^2-\Delta}}{m^2-\Delta}\right]^{1/2}-1\right],\\
C_{AB} &= \frac{\sigma^2}{4}C_{BA}=\frac{\sigma}{4}\sqrt{\frac{2m}{\Delta}}\left[\frac{m\mp\sqrt{m^2-\Delta}}{m^2-\Delta}\right]^{1/2}.
\end{aligned} \tag{2.14}$$

This case has been dealt with in [12] within an approach whereby the coefficients are defined as a series whose terms are obtained iteratively after fixing arbitrarily the freedom in the mapping which we have explained above. A generalization of the iterative mappings provided in that work may be retrieved from our procedure defining conveniently the operator σ (or $\hat{\sigma}$). By expressing

$$\sigma=\sum_{n=0}^{\infty}C_n\left(\frac{\Delta N_AN_B}{N_{AB}^2}\right)^n, \tag{2.15}$$

and expanding the exact mapping (2.14) in powers of Δ a series of increasing powers of Δ is easily obtained. In this setting the operator σ will give rise to a new independent parameter C_n at each new order of the series expansion. Indeed the series obtained iteratively in [12] corresponds to a particular choice of such set of constants.

On the other hand the operators defining the mapping can be alternatively expanded in powers of N_{AB} instead of powers of $1/N_{AB}$. Let us first consider the particularly important case corresponding to the limit $N_{AB} \longrightarrow 0$ in equations (2.10). This leads to the mapping from the fields of a model without topological terms to the purely topological model fields:

$$\widehat{C}_{AA} = \frac{2(-1)^{D(D-p)+1}\sqrt{\epsilon}\,[-N_A N_B \Delta]^{1/4}}{\hat{N}_A B^{1/2}\Delta\hat{\sigma}}, \qquad \widehat{C}_{BB} = \frac{(-1)^{D(D-p)+1)} N_A \hat{\sigma}}{2\hat{N}_{AB}^{1/2}\Delta[-N_A N_B \Delta]^{1/4}},$$
$$\widehat{C}_{AB} = \frac{2N_B}{\hat{\sigma}\hat{N}_{AB}^{1/2}\sqrt{\epsilon}\,[-N_A N_B \Delta]^{1/4}}, \qquad \widehat{C}_{BA} = \frac{\hat{\sigma}\sqrt{\epsilon}\,[-N_A N_B \Delta]^{1/4}}{2\hat{N}_{AB}^{1/2}}. \tag{2.16}$$

The inverse of this mapping may be directly obtained. This inverse mapping can also be obtained performing the limit $N_{AB} \longrightarrow 0$ already in the structure functions (2.8). A subtle technical point is that the main contribution that goes with $1/\Delta$ should be eliminated by first reabsorbing in equations (2.5) the first terms of the right-hand side of those equations. This amounts to a change in the gauge fixing conditions.

This limiting case can also be interpreted as the zeroth order term of the alternative expressions in powers of N_{AB} of the coefficients given in (2.10). Indeed the direct alternative power series expansion of (2.10) turn out to result in a set of series in powers of $N_{AB}/\sqrt{-\Delta}$ multiplying its zeroth order expressions (2.16). These series are essencially the same ones that have been made explicit in [1] in the four-dimensional case.

These series can be alternatively obtained in a procedure that parallels the one used to obtain the iterative mapping in powers of $\Delta\big/N_{AB}^2$. For this (2.16) should be taken as the zeroth order expression, that maps the action $\mathcal{S}_M(A) + \mathcal{S}_H(B)$ to $\mathcal{S}_{BF}(\hat{A}, \hat{B})$. Now the perturbation $\mathcal{S}_{BF}(A, B)$ is taken into account and its contribution is cancelled at each step of the process with higher order terms. Comparing to the previous procedures the roles of the kinetic terms are thus reversed. The presence of $\sqrt{-\Delta}$ in these series may seen suspicious at first sight. After all if the propagators of the Cremmer–Sherk model fields are expanded in powers of N_{AB} they turn out to produce, instead, a series of powers of $N_{AB}^2\big/\Delta$. Indeed an explicit computation shows that when the series obtained by expanding (2.8) is used to obtain the propagators of A and B from the ones of $\hat{A}$ and $\hat{B}$ the terms with square roots of the D'Alembertian cancel out.

Note that the gauge symmetry of the actions can be expressed using the $(n-p-2)$-form a and the $(p-1)$-form b as

$$\delta^g A = da, \qquad \delta^g B = 0 \tag{2.17}$$

and

$$\delta^t A = 0, \qquad \delta^t B = db. \tag{2.18}$$

while for the topological model fields we have the symmetries

$$\delta^g \widehat{A} = d\widehat{a}, \qquad \delta^g \widehat{B} = 0 \tag{2.19}$$

and

$$\delta^t \widehat{A} = 0, \qquad \delta^t \widehat{B} = d\widehat{b}. \tag{2.20}$$

The mapping (2.5) has been chosen in such a way that one pair of fields differs from the other in terms of gauge invariant terms. With this the gauge transformations of one pair of fields is translated straightforwardly into the ones of the other pair. That is, this leads to the identification of $\widehat{a}$ and $\widehat{b}$ with a and b, respectively. Of course other choices of mappings can be done which would result in different relations among the gauge parameters of each model but which still map their actions and thereby their free propagators.

3. Chern–Simons field

Let us consider now, in $D = 4n - 1$, the generalized Chern–Simons action given by

$$S_{\mathrm{CS}} = \int A \wedge \hat{\mathcal{D}}\, \mathrm{d}A. \tag{3.1}$$

Consider also the generalized Maxwell–Chern–Simons action

$$S_{\mathrm{MCS}} = \int (A \wedge \mathrm{d}\mathcal{D}A + F\mathcal{C} \wedge *F)\,. \tag{3.2}$$

Here the factors $\mathcal{C}$ and $\mathcal{D}$ are generic scalar operators that obey the same rule as Δ in equation (2.7).

We are searching for a mapping from the field $\hat{A}$ with pure Chern–Simons action to the Maxwell–Chern–Simons field A in the form

$$\hat{A} = -f * \mathrm{d}A + g * d * \mathrm{d}A. \tag{3.3}$$

Using this expression in the pure Chern–Simons action and requiring the outcome of the Maxwell–Chern–Simons action gives the set of equations

$$-fg\Delta = \frac{\mathcal{C}}{2\hat{\mathcal{D}}}, \qquad f^2\Delta + g^2\Delta^2 = \frac{\mathcal{D}}{\hat{\mathcal{D}}}. \tag{3.4}$$

The mapping becomes then

$$\begin{aligned} f &= -\left(\frac{\mathcal{C}}{2\hat{\mathcal{D}}}\right)^{1/2}\left[\frac{\mathcal{D}}{\mathcal{C}} + \epsilon\left(\frac{\mathcal{D}^2}{\mathcal{C}^2} - \Delta\right)^{1/2}\right]^{-1/2}, \\ g &= \left(\frac{\mathcal{C}}{2\hat{\mathcal{D}}}\right)^{1/2}\frac{1}{\Delta}\left[\frac{\mathcal{D}}{\mathcal{C}} + \epsilon\left(\frac{\mathcal{D}^2}{\mathcal{C}^2} - \Delta\right)^{1/2}\right]^{1/2}, \end{aligned} \tag{3.5}$$

where $\epsilon = \pm 1$ controls the different branches of the square root.

The mapping can be inverted by setting

$$A = -\hat{f} * d\hat{A} + \hat{g} * d * d\hat{A} \tag{3.6}$$

and using this expression in (3.3) and (3.5). The new structure functions turn out to be given by

$$\hat{f}g + \hat{g}f = 0, \qquad \hat{g}g\Delta^2 + \hat{f}f\Delta = 1. \tag{3.7}$$

The solution gives the inverse mapping coefficients as

$$\begin{aligned} \hat{f} &= \epsilon\left(\frac{\hat{\mathcal{D}}}{2\mathcal{C}}\right)^{1/2}\left[\frac{\frac{\mathcal{D}}{\mathcal{C}} - \epsilon\left(\frac{\mathcal{D}^2}{\mathcal{C}^2} - \Delta\right)^{1/2}}{\Delta\left(\frac{\mathcal{D}^2}{\mathcal{C}^2} - \Delta\right)}\right]^{1/2}, \\ \hat{g} &= -\epsilon\left(\frac{\hat{\mathcal{D}}}{2\mathcal{C}}\right)^{1/2}\frac{1}{\Delta}\left[\frac{\frac{\mathcal{D}}{\mathcal{C}} + \epsilon\left(\frac{\mathcal{D}^2}{\mathcal{C}^2} - \Delta\right)^{1/2}}{\frac{\mathcal{D}^2}{\mathcal{C}^2} - \Delta}\right]^{1/2}. \end{aligned} \tag{3.8}$$

Let us consider some limiting cases of interest. The case where $\mathcal{C} = 1/2 = \mathcal{D}/m$ corresponds to the mapping from the usual Maxwell–Chern–Simons model to the Chern–Simons model. This case in $D = 3$ has been dealt with in [1]. Second, let us consider the limit where $\mathcal{D} \longrightarrow 0$. When $\mathcal{C} = \hat{\mathcal{D}} = 1/2$ this limit corresponds to the pure Maxwell field action. In that case we have

$$\hat{g} = \sqrt{\frac{\hat{\mathcal{D}}}{2\mathcal{C}}}(-\Delta)^{-\frac{5}{4}}, \qquad \hat{f} = \sqrt{\frac{\hat{\mathcal{D}}}{2\mathcal{C}}}(-\Delta)^{-\frac{3}{4}}. \tag{3.9}$$

Another limit of particular interest is given by taking $\mathcal{C} \longrightarrow 0$, while $\mathcal{D} = \hat{\mathcal{D}} = 1/2$. This means to perform the limit from pure Chern–Simons to pure Chern–Simons models. The trivial mapping $\hat{A} \equiv A$ due to gauge freedom can be expressed in the setting of the expression (3.6) with

$$\hat{g} = 1/\Delta, \qquad \hat{f} = 0. \tag{3.10}$$

It is interesting to point out that besides this trivial mapping, by choosing conveniently the square root signals, the alternative mapping given by $\hat{g} = 0$ and

$$\hat{f} = \left(\frac{-\hat{\mathcal{D}}}{\mathcal{D}}\right)^{1/2} \left(\frac{1}{-\Delta}\right)^{1/2} \tag{3.11}$$

can be obtained.

The two limiting cases above may be considered as the zeroth order approximation of the two alternative iterative representations of the mappings. Let us first present the general form for the expansion in powers of $(-\Delta\mathcal{C}^2/\mathcal{D}^2)$.

$$\hat{g} = \frac{1}{\Delta}\left[1 + \frac{3\Delta\mathcal{C}^2}{2\mathcal{D}^2} + \frac{35\Delta^2\mathcal{C}^4}{128\mathcal{D}^4} + \frac{231\Delta^3\mathcal{C}^6}{1024\mathcal{D}^6} + \frac{6435\Delta^4\mathcal{C}^8}{32960\mathcal{D}^8} + \cdots\right] \tag{3.12}$$

$$\hat{f} = \frac{-\mathcal{C}}{2\mathcal{D}}\left[1 + \frac{5\Delta\mathcal{C}^2}{2^3\mathcal{D}^2} + \frac{63\Delta^2\mathcal{C}^4}{2^7\mathcal{D}^4} + \frac{429\Delta^3\mathcal{C}^6}{2^{10}\mathcal{D}^6} + \cdots\right]. \tag{3.13}$$

This series has been obtained in [3] in the $D = 3$ case through an iterative procedure. The exact mapping displays the non-local feature of the structure functions which may be somewhat masked in the direct iterative procedure.

Next let us present the series expansion in powers of $\mathcal{D}/\sqrt{-\Delta}\mathcal{C}$:

$$\hat{g} = \sqrt{\frac{\hat{\mathcal{D}}}{2\mathcal{C}}}\,(-\Delta)^{-\frac{5}{4}}\left[1 + \frac{1}{2}\frac{\mathcal{D}}{\mathcal{C}\sqrt{-\Delta}} - \frac{3}{8}\left(\frac{\mathcal{D}}{\mathcal{C}\sqrt{-\Delta}}\right)^2 - \frac{5}{16}\left(\frac{\mathcal{D}}{\mathcal{C}\sqrt{-\Delta}}\right)^3 + \cdots\right] \tag{3.15}$$

and

$$\hat{f} = \sqrt{\frac{\hat{\mathcal{D}}}{2\mathcal{C}}}\,(-\Delta)^{-\frac{3}{4}}\left[1 - \frac{1}{2}\frac{\mathcal{D}}{\mathcal{C}\sqrt{-\Delta}} - \frac{3}{8}\left(\frac{\mathcal{D}}{\mathcal{C}\sqrt{-\Delta}}\right)^2 + \frac{5}{16}\left(\frac{\mathcal{D}}{\mathcal{C}\sqrt{-\Delta}}\right)^3 + \cdots\right]. \tag{3.15}$$

This series expression can be derived starting from the zeroth order mapping pointed out above from the pure Maxwell to the pure Chern–Simons model and incorporating the Chern–Simons term iteratively.

Note the interesting property that although the mapping of the field involves odd and even powers of the parameter $\mathcal{C}/\mathcal{D}\sqrt{-\Delta}$ when computing the correlation functions for the A field using the series expansion only even powers of the parameter persist. Indeed this comes out since the series for $\hat{f}$ and $\hat{g}$ differ, essentially, for the signals of the odd order terms.

4. Conclusions and discussions

In this work we have studied the procedure that allows us to map the Maxwell–Chern–Simons field to the pure Chern–Simons field in $(4n-1)$D and the Cremmer–Sherk model to the Abelian version of the BF model in arbitrary dimensions. An striking importance of these mappings stems from the fact that the dynamical mass mechanism, which occurs in the topologically massive models, is allowed to be described in the context of purely topological models. The latter models presenting physical contents remarkably distinct from the former ones may offer new insights in this physical mechanism.

It is also remarkable that the mappings above presented allow for obtaining the time-ordered functions of one model in terms of the time-ordered functions of the other model. In this sense the mapping establishes not only a classical relation among the models but also a relation in the quantum sense. The possibility of obtaining the Green functions of the topologically massive models from the ones of the topological models which present scale invariance may offer valuable computational advantages. Nevertheless it is important to keep in mind that the possibility of obtaining the correlation functions of one model from a non-local mapping of the correlation functions of the other model does not imply the equivalence of the Hilbert space of both models.

The BF mapping has been established within an exact general procedure. One remarkable aspect that emerges is the presence of a great deal of freedom in this mapping. This freedom has been elucidated as in the four-dimensional case [1] as due to the form of the purely topological action which is defined through mixed products of fields. The invariance under rescaling of the fields of the BF-type action is responsible for it.

Although the topological terms are not required to establish the mappings we discuss here the topologically massive cases allowing for natural series expansions of the exact mappings. Indeed we presented two forms of series expansions which typically can be classified as infrared and ultraviolet series. The knowledge of the exact mappings provides us with a typical scale, given by the topological mass parameter. The two types of mapping shown here can be used for instance for computing loop variables of the generalized Cremmer–Sherk (or Maxwell–Chern–Simons) model using the corresponding expressions of the pure BF (or Chern–Simons) models. This suggests to perform the computation in closed fashion without resource to expansions given by the iterative mapping. In any case the mass parameter may provide valuable hints to discern in which cases computations using the usual iterative mapping [12, 13] should or should not be considered reliable. It can even provide alternative expansions for instance in direct powers of the mass parameter instead of the inverse power series. Let us also comment that it is to be expected [5] that the introduction of arbitrary gauge invariant interaction terms can be absorbed by considering nonlinear mappings.

In order to properly appreciate the physical meaning of the mapping, it is important to call attention to the necessity of defining the physical content of a local field theory in terms of the local polynomial algebra of observable fields. The mapping here provided relates two local models each with its physical Hilbert space reconstructed from the Wightmann functions of its own polynomial algebra [14, 15]. Since the mapping involves non-local functions it should be clear that within the pure BF model there are two Hilbert spaces to be obtained. One Hilbert space is obtained from the local polynomial algebra of fields defined after expressing the Cremmer–Sherk fields non-locally in terms of the pure BF model fields and it should not be confused with the other one, the Hilbert space of the pure BF model itself. This later is obtained from its local polynomial algebra of fields. Although constructed with the same model fields the first Hilbert space is not isomorphic to the second one. Instead, it will be isomorphic to the Hilbert space of the Cremmer–Sherk model. The same reasoning goes in the other direction of the mapping. In this context it is clear that neither Hilbert space should be considered as a subspace of the other. It is not a mapping of physical states that is being addressed here but a non-local mapping among the fields.

Acknowledgments

We acknowledge Faperj, FUNCEFETES and UVV for partial financial support.

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